Nuffield Seminar Series on Mathematical Knowledge in Teaching
Seminar 4: The case of argumentation and proof (Cambridge, 10th January, 2008)

Short input on

Stylianides, A. J., & Ball, D. L. (in press). Understanding and describing mathematical knowledge for teaching: Knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education.*

by Andreas Stylianides, University of Oxford

This commentary focuses on some general issues that arise from the paper and that relate to mathematical knowledge for teaching.

The paper is situated in the domain of research that aims to address this overarching question:

> *What mathematical knowledge can enable teachers to effectively support student learning of mathematics?*

Mathematics teacher educators in different institutions have developed, primarily for the purposes of their own teaching, courses that reflect (to some extent) their conceptions of what mathematical knowledge is important for teaching. However, there is often significant variation in these conceptions. This is not to blame mathematics teacher educators, but rather to raise the issue of how research can help mathematics teacher educators make informed decisions about what mathematical knowledge is important for them to promote in their courses.

The paper describes a two-step process for addressing this issue. The first step concerns the generation of hypotheses about what mathematical knowledge is important for teaching, and the second concerns the testing of these hypotheses, thus feeding back onto the first step. The paper focuses on the first step in the process, laying out and applying in the particular domain of proof an approach for generating hypotheses about important mathematical knowledge for teaching.

This approach involves the combined consideration of three factors:

1. Existing research and theory;
2. Mathematical analysis of successful teaching practices to examine the mathematical demands that are placed on teachers' knowledge as they support student learning in classrooms; and
3. Mathematical decomposition of the topics or concepts under consideration.

The emphasis on mathematics derives from the fact that we are interested in *mathematical* knowledge. The focus on teaching practice derives from the fact that we are interested in knowledge *for teaching*, that is, knowledge that would allow teachers to function effectively in classroom settings.

The hypotheses set forth in the paper with regard to knowledge about proof for teaching were derived from the combined consideration of these three factors. Specifically, consideration of existing research (cf. factor #1) revealed an emphasis on elements of knowledge about proof that relate to the logico-linguistic structure of proof. These elements of knowledge appear to be necessary for effective teaching, but, at the same time, consideration of teaching practice (cf. factor #2) indicated that these elements are inadequate to capture the complex nature of the knowledge about proof used by successful teachers *in action*. The central role of mathematical tasks in classroom activity that is documented in the literature (cf. factor #1) together with consideration of teaching practice that focused on proof (cf. factor #2) drew our attention in the paper to the complex relationship between proving tasks and proving activity that can be provoked by these tasks when implemented in classroom settings. Consideration of factors #2 and #3 enabled us to articulate some aspects of this complex relationship, thus offering insight into what else might be important for teachers to know about proof in addition to knowledge about the logico-linguistic structure of proof already identified in the literature.

The paper does not proceed to test the formulated hypotheses. This testing would involve investigation of the effect of the specified elements of knowledge about proof on (1) the quality of mathematics teaching, and (2) the quality of student learning about proof that such teaching would support. The presence of many confounding variables makes the task extremely complicated and methodologically challenging, but certainly not impossible.

I conclude by proposing three related issues for discussion.

1. What might be different approaches to deciding what mathematical knowledge in general, or knowledge about proof in particular, is important for teaching?

    (The two-step process that I outlined earlier is demanding and, even if we completed both steps in the particular domain of proof, it would be questionable whether the field could afford the time and resources to do the same for many other important mathematical topics and concepts. I am wondering whether, in the long run, we would be able to construct models that would simplify the process by allowing us to have enough confidence in our hypotheses from step #1, without the need for immediate and rigorous testing of these hypotheses in step #2 before they can inform teacher education and professional development programs.)

2. To what extent and in what ways does the mathematical knowledge about proof that is important for teaching depend on (1) the school level (primary/secondary), and (2) the teaching experience of teachers?

    (Background information: the paper was originally focusing on the primary school years but the editor made the point that the scope of the paper could be broadened.)

3. Given the constraints on how many different concepts or topics it is possible to cover in teacher education and professional development programs, how might mathematics teacher educators *prioritise* the different elements of mathematical knowledge that is important for teaching? And in particular: What place does proof deserve in this priority list?