

Nuffield Seminar Series on Mathematical Knowledge in Teaching

Seminar 4: The case of argumentation and proof (Cambridge, 10th January, 2008)

Stylianides, A. J., & Ball, D. L. (in press). Understanding and describing mathematical knowledge for teaching: Knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education*.

Short input by Cathy Smith, University of Cambridge

Andreas Stylianides was present to give us his perspective on this Stylianides and Ball paper so I shall refer to it as his paper throughout. The paper contains a careful analysis of proof within education, a proposed framework for understanding the relationship between proving tasks and proving activity, and an application of this to two episodes.

Points from the paper that stuck with me were:

1. the terminology of “proof” as a ‘valid’ proof (but with validity discussed) and “proving” as an activity that may or may not involve students producing a proof.
2. the framework for classifying tasks

the breakdown of knowledge to be mobilised in teaching as 1) knowledge of different kinds of proving tasks, and 2) knowledge of relationships between proving task and proving activity. My comments are drawn from my presentation and the discussion on the day: I found this distinction hard to sustain when reading the paper, in that it required me to hold ‘proving’ separate from constructing a valid proof. In the seminar we talked about proof implying closure and success, and related this to an earlier discussion of the distinctions made between reasoning, argumentation, and proof. I recall Julie Ryan’s suggestion of “argument-in-discussion”. We touched on, but left implicit, questions of when and how teachers might decide an activity is a proving activity. This seems to concern the curriculum as well as teacher knowledge. The framework for classifying tasks was proposed, and illustrated on the data given, and offered - hypothetically - as being generally useful. Andreas raised an interesting research question, which we didn’t discuss, about whether it was possible to ‘test’ such frameworks without extensive classroom trials. I considered how the framework was useful to my thinking about experiences of teaching proof. The two dimensions of the classification framework were number of cases (one/ finite/ infinite), and purpose, ie to prove or refute. Given how a task is placed on these dimensions, certain proving strategies are possible eg to offer a counter-example. My first response was that I didn’t particularly see why these two dimensions should be prioritised from a mathematical perspective. I think that as a secondary/ undergraduate teacher I take this knowledge about strategies for granted in my thinking about teaching. I would hope that all maths teachers did, but I suspect colleagues would tell me that many don’t. Not only that, but I would then want to develop all the connections eg between one example and the general case using the idea of a generic example; or between moving in Cabri through a sequence of a finite number of examples to illustrate the continuous variation in the infinite case; or between statements of a claim and its negation. In the seminar we discussed a resource which showed all 6 variants of a basic task (one/ finite/ infinite cases x prove/ refute), and considered: would this be useful to teachers to have? Or for students to produce? The dimensions may not be mathematically compelling, but they do have a simplicity that may be practical. I am

aware of recent work done by Nrich at Cambridge to classify their problem-solving task bank as opportunities for certain strategies, eg systematising, generalising, visualising. A difference seems to be that the Nrich classification is done *for* teachers whereas Andreas's paper seemed to envisage classifying as done *by* teachers as part of planning. We may well need both kinds of framework. An emphasis on tasks over a finite domain does seem to me to make the framework applicable to proving tasks in the classroom simply because many tasks used are of this form, in secondary as well as primary school. I suppose this is because they lend a goal and an accessibility to the lesson which supports pupils' engagement and teachers' time management.

The questions I was interested in are:

- We explored the idea of broadening the task from a finite number of cases to an infinite number. Are there any relationships we can exploit for teaching between **strategies** for enumerating a finite number of solutions to **strategies for generalising**?

I feel it is important for teachers to use equivalent statements of a problem: eg to say "why not others" as well as "why only these"? I was taught to ask – "What if not"? But what prompts the teacher to change, and change back? When does it become confusing?

There were some aspects of the teacher's strategy that I considered relevant that you didn't discuss. Particularly those concerning the interplay between mathematical activity, emotions and classroom management. For example, I understood that the teacher's **representation** with tallies and circles was linked to divisibility by 2 and had been used earlier by the students. But equally the students used final digits so could have used a - valid - exhaustive search mod 10 based on a working definition of even as 'ending in 0,2,4,6,8'. What makes the teacher go for one rather than the other? Is it just brevity? I think there is also a sense of seeing the problem a different way. In Vinner's terms, of getting the concept image getting closer to concept definition. And of introducing students to the canon of explanatory representations. Most – all - proofs leave something unsaid. The paper made me think about the grounds on which the teacher accepts that a student has a valid proof, and wonder whether a mathematical framework was all that was necessary. How does the teacher's scheduling of student contributions or subtask contribute to the students finding a valid proof? Or maybe I should instead think of how the classroom practice awards validity to a proof. In the second example given, the teacher saw a difference between one student judged to be recapitulating the representation in a particular case, and another as having a proof? Despite aiming to move away from logico-linguistic analysis I felt this paper's discussion rather relied on the 'text' of the proof as the basis for teacher's judgements. Andreas's 2007 paper in *ESM* tackles the same episode rather nicely, I think, concluding that Betsy does not have a 'proof' because other students are not able to follow it, and then discusses what the teacher's role might be.

In the same vein, I was curious about the discussion of the teacher's decision to work on a proposed counterexample before a proposed proof. On one level, this does illustrate that teachers will need to know that if a counterexample is accurate then a proof will be flawed and vv. On another, deciding which to tackle first brings in considerations about

timing and about students' emotional and behavioural reactions to the goals and complexity of the subtasks. I am starting to wonder whether a really useful classification would be one that teachers and students would understand as 'what kind of an ending is this maths lesson going to have?'