The need for a programme of research on educative curriculum materials as a mechanism for the diffusion/development of mathematical knowledge in/for teaching

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This paper argues that current political and professional thinking about mathematical knowledge in teaching reflects too restricted a conceptualisation of the issue: one which locates mathematical expertise within teachers alone, in isolation from the didactical tools in classroom use. Rather, this paper will suggest that the quality of mathematical knowledge and thinking achieved in everyday classroom practice is shaped by the design of tools as much as the action of teachers (recognising that there is, of course, interaction between the two). To develop this argument in more concrete terms, and bring out its relevance to current concerns of policy and practice, this paper will use the interim report of the Williams Review of Primary Mathematics Teaching (Williams, 2008) as a foil. Indeed, when this report draws attention to the "stark contrast" between teaching and "other professions, including medicine, law and engineering" (§32), one key aspect is the lack of recognition in relation to the teaching profession of the important part played by tools in mediating communal expertise. Accordingly, this paper concludes by proposing a programme of research guided by this broader conceptualisation, with a focus on practical strategies to capitalise on the potential of didactical tools to help bring mathematical knowledge to bear in teaching, and to develop practitioners' mathematical knowledge for teaching.

The interim report of the Williams Review draws on a predominant conceptualisation of mathematical knowledge in teaching. The report itself pinpoints the emphasis which this conceptualisation places on the teacher:

> It may appear idiosyncratic to come so late in this interim account of the review to the twin questions of curriculum and pedagogy. But, as outlined in the introduction, it is done deliberately and it is appropriate, again, to reiterate the views on the critical importance of the teacher, over and above all other factors. (§149)

However, the report does not embrace the widely-held misconception that any form of mathematical ability will be of professional value to the teacher. Indeed, it points to evidence which calls into question the assumed value of more advanced study of mathematics:

> It would be a mistake simply to equate specialist knowledge of mathematics alone with excellent teaching at this level. Indeed, a 1997 study for the (then) Teacher Training Agency (TTA) found that having an A-level in mathematics was not associated with effective teaching (as measured by higher gains in pupils' attainment). (§13)

Consequently, the report acknowledges that mathematical expertise relevant to teaching relates much more specifically to subject matter and subject pedagogy:

> In terms of meeting a school's priorities, respondents who were subject leaders or head teachers who taught mathematics highlighted the need both for improved general mathematics subject knowledge and for general mathematics subject pedagogy. (§50)

Hence, the report recognises how expert teaching depends on several forms of mathematically-related knowledge being drawn together:

> Ideally, ITT and CPD should provide opportunities for teachers to link deep mathematical knowledge with understanding of mathematical learning gained from research, diagnostic skills in observing children's learning, and the critical evaluation of pedagogic resources

> and approaches. (§182)

As the close of this last quotation suggests, the report treats didactical tools as subsidiary to teacher expertise. This is also illustrated in earlier discussion of the depth of understanding, and particularly capacity to make connections between cognate concepts and strategies, required of the teacher:

> To teach mathematics in a properly connected manner, teachers require deep curriculum knowledge. This should certainly extend beyond the KS2 curriculum, but as already discussed, may not need to go beyond GCSE. What is more important even than the extent of knowledge or competence is that the mathematics is understood in depth. For example, it is important that the teacher can see connections between fractions as parts of a whole, fractions as numbers on the number line, fractions as ratios, divisions, proportions in geometry, etc. (§180)

There is no recognition here, of the contribution that well designed curriculum materials can make to structuring classroom work so as to systematically bring out the connections between fractions in different guises, and to produce feedback bearing on students' grasp of these key connections. Equally, there is no recognition of the ways in which such materials (and accompanying pedagogical guidance) might support teachers' personal development and active deployment of such knowledge. In later discussion, however, the report does allude to the structure for classroom practice provided by the Mathematics Framework:

> … the original Primary Frameworks brought very considerable support to the classroom teacher. Indeed, visits show many classrooms in which these frameworks continue to form the bedrock of primary pedagogy. (§188)

Nevertheless, even here, there is no explicit recognition of how the value of such resources might move beyond support for teachers' lesson planning to development of their subject-related knowledge and thinking.

There is, however, implicit recognition of such potential in several of the criticisms that the report makes of the Framework. First, the report suggests that the prescriptive character of the Framework's 'three-part lesson' was not conducive to helping teachers develop depth and flexibility in subject pedagogy:

> The allocation of time and the 'pace' of lessons… need to be flexible enough to allow for different kinds of interaction and activity (whole class, pairs, groups, individuals) and for different timescales for each of these. For example, there should be scope for children to engage in extended problem-solving activity without this being artificially interrupted by the need to close the lesson with a plenary. (§176)

Likewise, the report criticises the Framework for failing to bring out important connections between mathematical ideas, and to organise topics coherently:

> … curriculum content must be presented in ways that emphasise the connections between mathematical ideas; mathematics is hierarchical, but not necessarily (in fact rarely) linear. However, one effect of the presentation used in the original frameworks was to compartmentalise the curriculum, and then to combine topics in a rather arbitrary way to construct two-week segments. (§177)

Finally, the report criticises the lack of an explicit model of pupil learning within the Framework's pedagogical guidance:

> Any meaningful discussion of pedagogy also needs to be based in a model of learning. The notes provided by the National Strategies about pedagogy fail to do this explicitly, but implicitly appear to adopt a broadly constructivist view… an approach the review supports. (§170)

Alongside concerns about the accessibility to teachers of the revised version of the Framework, this history leads the report back to an emphasis on the teacher, and on the need for organised professional development:

> Once again, the review is compelled to stress the importance of ITT and CPD in all aspects of practitioner training and development, including pedagogy and, in this case, the effective use of the Frameworks. (§189)

Nevertheless, these criticisms also point to the potential for curricular and pedagogical resources to be designed so as to fulfil an educative function for the teacher. Indeed, such resources could provide a highly effective means of supporting and bridging between the various forms of out-of-school and in-school professional development envisaged in the report, and the ongoing professional learning of teachers through everyday planning and reflection.

In a seminal paper, Ball & Cohen (1996) argue that curriculum materials could become more central to teacher learning if they moved beyond simple presentation of content to give more central attention to enacting curriculum in the classroom. For example, classroom resources could be designed to elicit key aspects of students' thinking for deeper attention. Pedagogical guidance could help teachers to learn how to interpret such thinking and respond to it. Likewise, classroom materials could deliberately incorporate alternative representations of mathematical ideas, focus attention on connections between them, and invite comparisons of them. Finally, while curriculum materials are often carefully designed, the accompanying pedagogical guidance is rarely designed to help teachers appreciate the didactical reasoning behind the design, and the mathematical knowledge underpinning it, so helping teachers to develop a deeper understanding of this knowledge, and enabling them to take this reasoning into account in selecting from the materials or modifying them. Subsequent research has identified some promise in using well designed curriculum materials to support teacher learning (Remillard, 2005).

More recently, Davis & Krajcik (2005) have developed this line of thinking by devising a set of design heuristics for educative curriculum materials in science education, capable of helping teachers develop more general knowledge that they can apply flexibly in new situations. Building from ideas about teacher learning, the heuristics relate to key components of a teacher's knowledge base: subject matter knowledge, pedagogical content knowledge for topics, and pedagogical content knowledge for disciplinary practices. Before attempting to overlay educative elements on existing curriculum materials, designers must ensure that these are well designed to promote student learning, and already informed by sound didactical thinking; regrettably, this rules out a high proportion of current curriculum materials. Pertinent also to the concerns expressed in the interim report of the Williams Review about the online versions of the Mathematics Framework, Davis & Krajcik identify the potential of hypermedia as a platform for educative curriculum materials: for example, in permitting greater personalisation of interaction, and stronger integration of textual and audiovisual material; but also as presenting potential difficulties through superficial navigation and information overload.

The main achievement of the work reported by Davis & Krajcik has been to operationalise the set of principles put forward by Ball & Cohen into a system of design heuristics which have been used to guide development of educative science curriculum materials. Elaborated more fully in their paper, the basic design heuristics are:

- Supporting Teachers in Engaging Students with Topic-Specific Scientific Phenomena
- Supporting Teachers in Using Scientific Instructional Representations
- Supporting Teachers in Anticipating, Understanding, and Dealing with Students' Ideas About Science
- Supporting Teachers in Engaging Students in Questions
- Supporting Teachers in Engaging Students With Collecting and Analyzing Data
- Supporting Teachers in Engaging Students in Designing Investigations
- Supporting Teachers in Engaging Students in Making Explanations Based on Evidence
- Supporting Teachers in Promoting Scientific Communication
- Supporting Teachers in the Development of Subject Matter Knowledge

There is scope, then, to seek to adapt this type of approach to the design of educative curriculum materials for mathematics education (at secondary-school level as much as primary).

In brief, then, my argument is that the interim report from the Williams Review has correctly recognised the enormous challenge of strengthening mathematical knowledge for teaching systemwide. Equally, whatever might be ideal, the report recognises that the scale of organised professional development required to address the challenge through that mechanism is not realistically achievable. Instead, it has settled for a more modest programme, much of it in-school, and dependent on the leadership of a cadre of specialist teachers. Finding and developing this cadre of specialist teachers itself represents a further challenge, as does devising effective forms of in-school professional development. Where my argument goes beyond that of the report is in suggesting the potential of educative curriculum materials to support and bridge between these various forms of professional development, and to embed professional learning in teachers' everyday work. This, I suggest, represents an important second prong of any strategy aimed at systemwide development of mathematical knowledge for teaching: first, it aims to ensure that the curriculum materials used by teachers are informed by the public corpus of mathematical knowledge for teaching already available within the community as a whole; second, it aims to ensure that such materials are designed so that everyday use – and planning for and reflection on that use – will support the diffusion of that public corpus of mathematical knowledge for teaching across the profession. This points to an important research programme, focusing on the development and testing of design heuristics for educative curriculum materials themselves, and for forms of professional development centred on their use.

References

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