Mathematical Knowledge in Teaching: the case of argumentation and proof
Cambridge, 10 January 2008 - Group 3 Discussion Summary

Our discussions began by considering a number of questions which arose from the presentation of the three papers, and put forward by the group facilitator.

1. What do we mean by proof?
   - Is this something we think teachers need to know about?
   - Is this the same for primary and secondary teachers?
   - Is there an appropriate framework that teachers could use?
2. How do we regard the collective knowledge generated by argumentation?
   - What do the teachers bring to the knowledge?
   - What do the pupils bring?
3. What is the role of tasks in teaching proof?
   - Do teachers need to have knowledge of types of tasks and proofs?
4. How do we establish in teachers (particularly primary) the notion of 'truth' in mathematics?

Having set out the questions and revisited the theme of the group discussion *'developing and deepening mathematical knowledge in teaching proof'* we discussed a variety of points which are summarised as follows.

- Primary teachers are very often not mathematics specialists, even if they hold positions of leadership, for example as mathematics subject leaders. In that case, with no specialised training in mathematics, it must be very difficult for them to have a notion of proof which they can use to teach effectively or to lead colleagues in. However, 'proof' as a method has appeared implicitly in various versions of the primary mathematics framework since the National Curriculum was introduced.
- The word 'proof' is a noun and as such it suggests perhaps the title of a session which can be taught (or delivered) and then ticked off as 'done'! Problem-solving in primary schools is often treated in a similar way. We felt there should be more of a move towards the idea of 'proving' as an activity or experience in relation to the learning of areas of mathematics, and that in ITE there should be good models given to students of what constitutes 'reasoning in mathematics'. There is a difference between learning mathematical content (e.g. for a test) and learning to reason (or prove), and mathematics educators should be steeped in knowledge of proof, and therefore well-placed to pass this on to students.
- Reasoning should be embedded in primary mathematics, and as part of ITE mathematics courses, the underlying concepts should be examined by example generation, for example.
- There is a danger in isolating reasoning to mathematics. Teachers could legitimately look for comparisons with reasoning from other subject disciplines, such as debates in history, and use common reasoning skills to help establish inquiry-based mathematics classrooms. Similarly, teachers should be looking to relate mathematical content to other subject areas in order to provide a relevant and connected learning experience. Otherwise, mathematics will continue to be seen as a very distinct area of learning where only 'right' and 'wrong' exist, although this may be perpetuated by the test-driven culture.
- Problem-solving often becomes a recipe to follow, and devoid of much meaningful mathematical content. Pupils need experience of the process of solving problems, but through tackling appropriate mathematical tasks.
- There is an enormity of shift between going through a process of task completion which requires discussion and reasoning and the written outcome often required in school. The shift may be impeding the reasoning process.
- Time and curricular constraints may be barriers to deepening use of proof.
- To address some of these issues, CPD should be used to develop knowledge of proof in teachers, and both ITE and CPD should make greater use of task-based learning in which proof and reasoning will naturally arise, and the mathematical discourse of proof can be given greater priority.

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