

CONCEPTUALISING TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING

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In the literature different analytic models of teachers' knowledge are proposed. Currently, there is no agreement on a widely accepted model that can be used to describe teachers' mathematical knowledge for teaching. Our aim is to discuss the different models of teachers' knowledge existing in the literature, and explain how these can be understood as elaborating Shulman's (1986) conceptualisation of content related categories of teachers' knowledge. Furthermore, this paper aims to show how the existing models on teachers' knowledge can be seen as a start in addressing concerns about whether the distinction between SMK and PCK could and should be made. We go on to suggest a number of research questions and explore the individualistic nature of conceptions of PCK.

Clearly, Shulman (1986) initiated a new wave of thinking about teachers' knowledge by suggesting that content should matter in teaching. The most important claim in his account is that both knowledge of the subject and knowledge of pedagogy are needed in effective teaching.

Fennema and Franke (1992) built on the work of Shulman by suggesting that knowledge needed for teaching is interactive and dynamic in nature, and research methodology needs to focus on understanding the interrelation between theoretical knowledge domains. The Knowledge Quartet (Rowland, Huckstep and Thwaites, 2005) responds to the work of Fennema and Franke by suggesting ways in which teachers' SMK relates to their PCK, and ways in which teachers' actions in the classroom are informed by different aspects of their knowledge. Finally, the Mathematics Teaching and Learning to Teach and the Learning Mathematics Teaching projects resulted in the conceptualisation of a model of teachers' knowledge, that can be understood as an attempt to validate Shulman's conceptualisation by developing reliable and valid measures of mathematical knowledge for teaching, and use these measures to test theoretical definitions and models suggested for the nature of teachers' knowledge (Ball, Thames and Phelps, 2007).

The different models of teachers' knowledge proposed in the literature will continue to need refinement and work. The categories of teachers' knowledge proposed in the models might not be the 'right' ones; probably they are not, since research in the field of teachers' knowledge has done little to clarify, and empirically test the theoretical conceptualisations proposed in the literature. However, whether the categories are the right ones is not most important (Ball et al, 2007). What is common and of great importance in all the accounts described above is that teachers' knowledge needed for mathematics teaching is multidimensional. Central is the idea that different aspects of teachers' knowledge, such as their knowledge of content and their knowledge of pedagogy are combined to each other to create a set of knowledge that determines teachers' behaviour in the classroom.

All the conceptualisations of teachers' knowledge mentioned can be seen as a start in addressing concerns about whether the distinction between SMK and PCK could and should be made. Indeed, McNamara (1991) raised the question, of whether or not it is possible to distinguish SMK from PCK. Aubrey (1997) and McEwan and Bull (1991) seemed to agree with McNamara (1991) and argued that there is no distinction between SMK and PCK and that all knowledge is pedagogic. Aubrey (1997) defines 'pedagogical subject knowledge' (PSK) as,

the superordinate category which subsumes the sub-category of subject-matter knowledge, on the one hand, and knowledge of young children's subject understanding on the other, and incorporates curriculum knowledge which, in providing the means to represent this subject

matter in ways which make sense to children, exemplifies teachers' pedagogical reasoning (Aubrey, 1997, p.164)

While these definitions reflect the idea that PCK combines the subject with teaching they include everything that teachers might need to teach mathematics, and they do not offer the opportunity to make a distinction between teachers' actions, knowledge and beliefs. They are so broad that it is unclear where boundaries might exist, if they do, between knowledge of content and pedagogy. Also, it is difficult to see the practical utility of a unidimensional conceptualisation of teachers' knowledge in attempts to improve teacher preparation programmes, because it would be very difficult to decide what aspects of teachers' knowledge to include. A clear sense of the categories of teachers' mathematical knowledge for teaching can help researchers identify which categories of mathematical knowledge for teaching are the greatest predictors of students' achievement or effective mathematical thinking (Ball et al, 2007). Also, a clearer understanding of what teachers' knowledge includes can help policy makers to develop more effective curriculum for teacher training programmes.

Whether we use PCK as an overarching category or continue to see SMK and PCK as dynamically related the frameworks for teacher knowledge are individualistic. They focus on the knowledge that an individual teacher brings to the classroom. In terms of teacher training, there is an assumption that we need to check the applicant's previous qualifications for suitability and in terms of secondary courses to address subject knowledge in some way at interview. During the training course the assumption seems to be that the SMK relevant to the phase that the trainee will be teaching will be unpacked and that PCK will be developed by the training course by elements of both university based and school based work including the use of assignments.

Research questions derived from these assumptions could be

- What previous qualifications do trainees hold?
- How is subject knowledge assessed at interview? What happens to the information elicited? Is it used to deselect applicants? Is it used to inform course planning or support for trainees?
- How is the development of PCK monitored and tracked during the training course?

The Shulman model has been used to inform the design of audit instruments to assess and remediate the mathematical knowledge for trainee teachers (Rowland, 2007). Refinements offer the promise of being able to identify which categories of mathematical knowledge for teaching are the best predictors of students' achievements and mathematical thinking (Ball et al., 2007). This knowledge may enable policy makers to design suitable curricula for teacher training programmes (Petrou, 2007). However, the stress on the individual can be seen as punitive, enabling the deselection of unsuitable teachers, or naming and shaming during the training course. Alternatively, there is evidence of individual assessment which has fed into productive peer learning (Barber, Heal and Martyn, 2002) and both peer and one to one work with tutors (Ryan and Williams, 2007).

Research questions

- There has been considerable research on trainees'/teachers' understanding of division and fractions relevant to primary and lower secondary teaching. What do we know about trainees'/teachers' knowledge of space and shape, reasoning and proof, and areas of secondary teaching e.g. algebra, calculus.
- How has knowledge gained from audits been used to influence the design of training opportunities? Do trainers focus on individual or groups in addressing subject knowledge weaknesses?
- How does self assessment from audits feed into the trainees'/teachers' preparation for teaching?

- Should we abandon audits?

The individual model of teacher knowledge has been used to analyse trainee teachers' classroom performance and to suggest ways in which this analysis can inform strategies to develop teacher knowledge in the primary phase (Turner and Rowland, 2008).

- Does the Knowledge Quartet framework apply in the secondary phase?
- The stimulated recall methodology used to research the use of the Knowledge Quartet enabled teachers to reflect on their mathematical knowledge in practice. Could this be extended to larger numbers of teachers in different settings in order to determine the conditions under which an individual teacher in a community of practice becomes a collective community of enquiry?

Challenges to the individualistic model stress the social nature of teachers' work. Once faced with real classes on teaching practice, trainees do not need to rely solely on what is in their own heads. They can access curriculum materials and research and can seek the help of other trainees and their school based mentors. The knowledge for teaching is distributed across material and human resources. Moreover, trainees may perform very differently when faced with real children in the context of the classroom rather than in a one to one interview (Hodgen, 2007).

- Do trainees/teachers identify areas of PCK in which they need support?
- How exactly do trainees/teachers use the resources for mathematical knowledge at their disposal to inform their planning and interactions with pupils?
- For those trainees/teachers who have weaknesses and do not seek support how is this handled within the school community?

Overarching research questions

- Are individual and social views of teachers' knowledge competing or can they be reconciled?
- Is there a way of integrating individual knowledge within a social framework?
- Is this a conceptual question or can we investigate it empirically?

Postscript

The tension between individual and distributed knowledge is not just a question for teachers. How we frame teacher knowledge may be related to our assumptions about students' knowledge. In the case of argumentation and proof, will an emphasis on the teacher's individual knowledge hinder the development of collective argumentation? What is it about the teacher's knowledge that facilitates the development of collective argumentation in the classroom?

References

- Aubrey, C. (1997) *Mathematics Teaching in the Early Years: An Investigation of Teachers' Subject Knowledge* (London, The Falmer Press).
- Ball, D. B, Thames M.H. and Phelps, G. (2007) Content knowledge for teaching: what makes it special?, Manuscript submitted for publication, <http://www-personal.umich.edu/~dball/papers/index.html>
- Barber, P., Heal, C. and Martyn, S. (2002) 'Primary teacher trainees' mathematics subject knowledge - progress in developing systems of support?' *Annual Conference of the British Educational Research Association*, University of Exeter, September.
- Fennema, E. and Franke, L. M (1992) Teachers' knowledge and its impact, in D.A. Grouws (Ed) *Handbook of research on Mathematics Teaching and Learning* (New York, Macmillan), 147-164.
- Hodgen, J. (2007) The situated nature of mathematics teacher knowledge, *Mathematics Knowledge in Teaching Seminar Series: Conceptualising and theorizing mathematical knowledge for teaching (Seminar 1)*, University of Cambridge, Cambridge.

- McEwan, H. and Bull, B. (1991) The pedagogic nature of subject matter knowledge, *American Educational Research Journal*, 28 (2), 316 - 334.
- McNamara, D. (1991) Subject knowledge and its application: problems and possibilities for teacher educators, *British Educational Research Journal*, 28(5), 113-128.
- Petrou, M. (2007) Michigan research on developing practiced- based theory of knowledge of teaching, *Mathematics Knowledge in Teaching Seminar Series: Auditing and assessing mathematical knowledge in teaching (Seminar 3)*, Nuffield Foundation, London.
- Rowland, T. (2007) Auditing the mathematics subject matter knowledge of elementary school teachers, *Mathematics Knowledge in Teaching Seminar Series: Auditing and assessing mathematical knowledge in teaching (Seminar 3)*, Nuffield Foundation, London.
- Rowland, T., Huckstep, P. and Thwaites, A. (2005) Elementary teachers' mathematics subject knowledge: the knowledge quartet and the case of Naomi, *Journal of Mathematics Teacher Education*, 8(3), 255-281.
- Ryan, J. and Williams, J. (2007) Mathsmaps for diagnostic assessment with pre-service teachers, *Mathematics Knowledge in Teaching Seminar Series: Auditing and assessing mathematical knowledge in teaching (Seminar 3)*, Nuffield Foundation, London.
- Shulman, L.S. (1986) Those who understand, knowledge growth in teaching, *Educational Researcher*, 15(2), 4-14.
- Turner, F. and Rowland, T. (2008) The Knowledge Quartet: a means of developing and deepening mathematical knowledge in teaching, *Mathematics Knowledge in Teaching Seminar Series: Developing and deepening mathematical knowledge in teaching (Seminar 5)*, Loughborough University, Loughborough.