Nuffield Seminar Series: Mathematical Knowledge in Teaching

Seminar 4: Argumentation and Proof

Yackel, E. (2002) What can we learn from analyzing the teacher's role in collective argumentation? *Journal of Mathematical Behaviour.* 21: 423-440.

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Preamble

The following summary represents our pre-seminar discussion but is also informed by the discussion in the seminar itself. Maria's contribution is on aspects of the theoretical framework, Marie's discusses the case studies presented by Yackel, and concludes with 'our' thoughts.

Theoretical Framework

Yackel, researching the teacher's role in the development of argumentation in the classroom, endorses the premise that

> 'Teachers can only create and capitalize on opportunities to pursue mathematical ideas and connections across the curriculum to the extent that they are able to recognize such opportunities'. (p. 439)

If we accept this relatively unproblematic statement then courses of teacher preparation and development need to address this aspect of mathematical teacher knowledge. However, the primary trainee teachers involved in the Subject Knowledge in Mathematics Project (SKIMA) research, were identified through subject knowledge audits as having weaknesses in reasoning and proof but did not themselves see it as a particular cause for concern (Goulding, Rowland & Barber, 2002; Goulding, 2002). This may have been because they did not see reasoning and proof as being integral to mathematics and hence something they would not have to address in primary classrooms. It is possible that this perception may have been created or exacerbated by an audit process which necessitated the successful completion of discrete items, which may have obscured the view of mathematical reasoning 'as central to what it means to engage in mathematical activity'. (National Council of Teachers of Mathematics, quoted in Yackel, 2002, p. 423).

In contrast, the practising teachers in Yackel's study were all engaged in experiments or development programmes aiming to foster not only mathematical reasoning but also collective argumentation. This movement from an individual to a social notion of reasoning is the backdrop to this research, as the focus is on the teacher's use or lack of use of language, action, diagrams and notation in productive or unproductive classroom interactions. There are echoes of the connections and contingency categories of the Cambridge team's knowledge quartet in Yackel's data when teachers either intervene productively or miss opportunities to capitalise on students' contributions to classroom discussion (Rowland, Huckstep & Thwaites, 2004).

One of the interesting features of discussion about mathematical reasoning is the inflection of meaning which can occur with small changes to the vocabulary. So the noun 'proof' has connotations of closure and rigour and conjures up an image of a disembodied object existing outside the material condition in which it was created. For those who experienced proof in school mathematics almost entirely through the study of Euclidean geometry the associations may be rather dry – some will remember the spatial rules for setting out the proofs of theorems long after forgetting the results which were derived. Others schooled in the same era remember the process of using these theorems to derive new results or riders as a more exciting and, in my case, a co-operative venture.

In recent research on the development of reasoning and proof non geometrical examples involving the properties of odd and even numbers have become very common in the literature. If we change the instruction ‘prove that the sum of two odd numbers is even’ to ‘argue that the sum of two odd numbers is even’ the use of a word which is perhaps more common in non mathematical conversations seems to emphasise the social process stressed by Yackel. Whilst recognising that importing everyday meanings into the classroom is problematic, the everyday notions of trying to defend one’s position or, better, to convince another person of its validity, is useful if we aim to induct pupils into a classroom culture which reflects the way in which mathematical proofs are constructed in the academic world. The word ‘argumentation’ does not seem to be common in mathematics curricula or mathematics education research, but it may have advantages if the social process of ‘proving’ is to be encouraged.

Yackel’s theoretical framework employs Toulmin’s argumentation scheme consisting of conclusions (or claims), data, warrants and backing. Like ‘argue’, the term ‘warrant’ may have useful connotations with guarantees and authority which may be helpful to both teachers and pupils. This is not the language of the mathematics National Curriculum in England and Wales, which uses terms such as ‘drawing inferences from data’ (PoS, KS1), ‘explain their methods or reasoning’, (PoS, KS1) ‘justify their generalisations, arguments or solutions showing some insight into the structure of the problem’ (AT1, level 7) . Nor is it the language of mathematics itself in which we would use terms such as ‘conjecture’, ‘inductive’, deductive’, ‘refutation’, ‘contradiction’, words which seem independent of the social process in which they occur.

Questions arising for both research and teacher preparation include

- Does a framework derived from philosophy help us analyse the process of argumentation illustrated in Yackel’s data?
- Are there more appropriate alternative frameworks? Ken Ruthven suggested that linguistic frameworks may be more helpful in addressing the question ‘How does one make conversations work in a multipersonal situation?’
- Does the language of mathematics and the National Curriculum in Mathematics support or hinder the collective process of mathematical argumentation?

We do not intend to answer these questions here, but in the section below we look at Yackel’s case studies and begin to answer the first question.

An analysis of the case studies

Yackel draws the first of her five case studies from the higher education sector, in a classroom where the tutor has established a culture of discussion. She describes some discussion between the students and points out the key role of the teacher in encouraging the discussion and in drawing out the mathematical ideas suggested by the students. However, the teacher does not evaluate the students’ ideas or their reasoning nor, using the language of argumentation, does he require the students to provide much more than claims and data.

In three further case studies taken from a primary (elementary) school setting, the teacher asks a student to explain what he has done or how he has come to his answer. In all these cases, the teacher’s role can be seen as analysing the students’ explanations in terms of the current understanding of the other students of the class, and providing further warrants and backings.

These case studies do not really provide examples of **collective** argumentation, in contrast to the third case study set in a primary school, where several children discuss and reason in order to agree the answer to a question. In the end, however, they use a calculator to provide 'proof' of the correct answer, and, as Yackel comments, there is perhaps a missed opportunity on the teacher's part. This, then, demonstrates how collective argumentation, without appropriate intervention on behalf of the teacher, does not always lead to 'good' mathematical reasoning.

A final example (again from a primary classroom) focuses on the teacher's own mathematical knowledge in developing a situation which calls for 'proof' (How can you be sure?) A student provides an explanation which the teacher did not perhaps expect, and Yackel points out that, without the confidence provided by the teacher's own mathematical knowledge, the teacher may not have been able to draw out and develop the explanation.

We question the extent to which the argumentation framework is of any value in analysing these case studies. It seems to us that what is really important here is the teacher's role in classrooms where mathematical reasoning takes place as part of classroom discussion. The suggestion in the second question above would seem to be a better way forward, since it integrates the mathematical aspects of proving with the pedagogical aspect of managing classroom talk. We recognise the complexity of this role and suggest that the mathematics education community uses research such as this to inform aspects of programmes of initial teacher education and continuing professional development.

References

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