MATHEMATICAL SUBJECT KNOWLEDGE IN PRIMARY TEACHER TRAINING – A VIEW FROM ENGLAND AND WALES

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Background

When professional standards for qualified teacher status were introduced in England and Wales in 1998 (Department for Education and Employment, Circular 4/98) there was a strong emphasis on trainee subject knowledge in the core subjects of English, mathematics and science. A list of mathematical topics, described as that which will 'underpin effective teaching' was prescribed. Institutions involved in teacher training were required to audit this knowledge and where gaps were found to fill them by the end of the training course.

The concern with subject knowledge, particularly for primary trainees, had been growing for some time. It had long been felt that strengthening primary teachers' subject knowledge would contribute to the aim of improving standards in mathematics (Alexander, Rose and Woodhead, 1992). The later evaluation of the first year of the National Numeracy Strategy (Ofsted 2000) located weaknesses in teachers' mathematical subject knowledge, particularly "teaching of progression from mental to written methods; problem solving techniques; and fractions, decimals and percentages" (p. 6). At this time, then, the audits and consequent remediation were considered to be a strategy for addressing this problem.

Interestingly, the process of auditing stimulated a flurry of associated research. For instance, there was considerable concern about trainees' anxiety and how best to address weaknesses in ways which would not damage self confidence (see Barber and Heal, 2003, for a discussion of peer tutoring). Murphy (2003) found a tension between trainees seeing the process as useful and confidence building or just a hoop to be jumped through. In terms of mathematics, insights into difficulties with geometry and probability (Mooney, Fletcher and Jones, 2003) and reasoning and proof (Goulding, Rowland and Barber, 2002) were found.

During this period, government inspectors claimed to find that trainees' mathematical subject knowledge had improved substantially, attributing this partly to more systematic and less superficial auditing of subject knowledge. The modified regulations (TTA, 2002a), which are at the time of writing still in place, did not specify a body of knowledge or require an audit, but one of the assessment standards is 'a secure knowledge and understanding of the subject(s) [the trainees] are trained to teach.' The non-statutory handbook (TTA, 2002b) suggests that the source of evidence for this standard 'is most likely to be found in trainees' teaching, particularly in how they present complex ideas, communicate subject knowledge, correct pupils' errors and in how confidently they answer subject-based questions' (part 1, para. 2.1, p11). It is for institutions to decide how to ensure that trainees have a sufficient grasp of 'the concepts, ideas and principles' they will need to be able to teach in the age range for which they are preparing.

Concern with teacher's knowledge of mathematics is not confined to England and Wales. In the US, both theoretical frameworks and empirical research on teacher knowledge are well developed and were highlighted by the National Council for Teachers of Mathematics, in several chapters of its extensive review of research in mathematics education (Grouws, 1992). Much of this research is premised on the belief that learning is a product of the interaction between what the learner is taught and what the learner brings to the learning situation. For Ball, the knowledge prospective teachers bring to their teacher education courses is framed negatively:

> This lack of attention to what teachers bring with them to learning to teach mathematics may help to account for why teacher education is often such a weak intervention - why teachers, in spite of courses and workshops, are most likely to teach math just as they were taught. (Ball, 1988, p. 40)

Conceptualising subject knowledge

Much of the research mentioned previously drew on Shulman's categories of teacher knowledge, and, in particular, his constructs of *subject matter knowledge*, *pedagogical content knowledge* and *curricular knowledge*. These classifications are generic but can be applied with some adjustment to the discipline of mathematics.

Subject matter knowledge is the 'amount and organization of the knowledge per se in the mind of the teacher' (Shulman, 1986, p9) and is later (Shulman and Grossman, 1988) further analysed into *substantive knowledge* (the key facts, concepts, principles and explanatory frameworks in a discipline) and *syntactic knowledge* (the rules of evidence and proof within that discipline). His *pedagogical content knowledge* consists of

> the most powerful analogies, illustrations, examples, explanations and demonstrations – in a word the ways of representing the subject which makes it comprehensible to others…[it] also includes an understanding of what makes the learning of specific topics easy or difficult … (Shulman, 1986, p9)

The boundaries between SMK and PCK may well be blurred; students who have multiple representations for mathematical ideas and whose mathematical knowledge is already richly linked will be able to draw upon these both in planning and spontaneous teaching interactions. In such cases it would seem that the students' subject knowledge is ripe for exploitation and that in turn the experience of teaching will feed back into and enrich subject matter knowledge.

Shulman's c*urricular knowledge* consists of knowledge of the scope and sequence of teaching programmes and the materials used in them. Again, there could be a feedback loop, as teachers encounter new ways of thinking about and representing mathematical ideas in textbooks and other resources.

Subsequent writing by Shulman and his associates (Grossman, Wilson and Shulman. 1989) takes into account student beliefs about mathematics, whilst Ernest (1989) identifies attitudes (interest and confidence) as important. Beliefs about the nature of mathematics may be tied up with SMK in the way in which teachers approach mathematical situations. If they believe that mathematics is principally a subject of rules and routines which have to be remembered, then their own approach to unfamiliar problems will be constrained, and this may have an impact on their teaching. Beliefs may be particularly salient in the development of syntactic knowledge, where conjecturing, finding evidence and seeking explanations is quite different from applying rules and routines in recognisable contexts. If teachers lack confidence in their SMK, then they may avoid risky situations in the classroom and be inhibited in responding to children's unexpected questions. They might also seek refuge by opting to teach younger children where they may feel less daunted by the demands of the mathematics curriculum, denying themselves the opportunity to engage with material which would challenge and develop their SMK. On the other hand, teachers who lack confidence may be more inclined to prepare their lessons carefully and to access a range of resource material. If teachers lack interest, their planning and preparation may be skimpy and they may convey feelings of negativity, or at best indifference, to their pupils. The myth that good SMK in mathematics is somehow a barrier to teaching the subject to younger pupils and young children and that those with weak SMK can compensate for this with empathy may be unhelpful.

Research on Teachers' Mathematics Subject Knowledge

Ball echoes Shulman's constructs of substantive and syntactic knowledge in any discipline by making a distinction between knowledge *of* mathematics (meanings underlying procedures) and knowledge *about* mathematics (what makes something true or reasonable in mathematics). Investigating both elementary and secondary pre-service teachers' understanding of division, she found (Ball, 1990a) that both had significant difficulties with the *meaning* of division by fractions. Most could do the calculations, but their explanations were rule-bound, with a reliance on memorising rather than conceptual understanding. They were also at a loss as to how they could justify answers. Although they wanted to be able to give pupils' meaningful explanations, their own limited subject knowledge would prevent them from being able to realize their aim. They believed that mathematics could be meaningful but lacked the knowledge of these meanings themselves.

Division by fractions was also the focus of a short account by Ng (1998), working with trainee teachers on illustrative diagrams which gave meaning to the answers obtained by using a division algorithm. Ma's (1999) comparative study of Chinese and US teachers' profound knowledge of 'fundamental' mathematics (PKFM), also investigated dividing fractions by fractions as one of four problem scenarios. Compared with the American teachers, who mostly had a limited and sometimes faulty understanding, the Chinese teachers were able to do the calculation, provide examples of the concept in context and discuss concepts underpinning the understanding of division. In other topics, they were also able to articulate salient mathematical structural properties e.g. distributivity to justify the standard procedure for 'long multiplication'. Ma concludes that no amount of general pedagogical can make up for ignorance of particular mathematical concepts.

In her earlier study, Ball (1988) explored the distinction between what mathematics should be known and how it should be organized. The 'what' tends to be listed, as in Circular 4/98, and the 'how' tends to be described qualitatively by words such as 'flexibility' and 'in-depth'. Her term 'connected' (Ball, 1990b), derived from comparison of the knowledge held by expert and novice teachers, has since been used by others as a way of describing the quality of subject knowledge that teachers need (e.g. Askew et al., 1997). So in the case of division, the knowledge displayed by Ma's Chinese teachers could be described as connected, or in her terminology 'profound'. Ng's approach with prospective primary teachers could be seen as a way of developing connected subject knowledge in training.

In their training, Ball argues that not only should mathematics be revisited, but also that pre-service teachers may also need to 'unlearn' what they know about teaching and learning of mathematics. Her programmes at Michigan State University are designed 'to surface and challenge' assumptions by requiring students to engage with some unfamiliar mathematics (e.g. permutations) themselves, observe her working with a young child exploring the concept and then take on the role of teacher in exploring the concept with someone else, not necessarily a school pupil. The reflections of the students on these courses clearly demonstrate the effect of this 'unlearning' on their self-awareness and their beliefs about mathematics. One student wrote:

> I'm learning about mathematics in this class…math isn't just about memorising formulas – it is knowing *why* a problem is done the way it is…In high school, [it was] memorising formulas, theorems and definitions. (Ball 1990b, pp. 43-44)

Research on the Relationship between Subject Knowledge and Classroom Teaching

If categorising teachers' knowledge is complex, then trying to explore it in action is even more difficult, quite apart from making judgements about the relationships between variables such as subject matter knowledge and pupil progress. Aubrey (1997) acknowledges this in her comprehensive review of the literature on competent teaching performance, largely derived from US comparisons of expert and novice teachers, but argues for 'the central importance of disciplinary knowledge to good elementary (primary) teaching' (p.33)

Aubrey's conceptualisation owes much to Shulman but she uses a superordinate category of pedagogical subject knowledge, incorporating SMK, knowledge about children's understanding and curricular knowledge. Her study set out to investigate teachers' pedagogical subject knowledge, the informal knowledge of pupils at school entry and their teachers' understanding of what their own pupils know and how they think about mathematics, in order to consider the implications for the mathematics curriculum in the first year of schooling.

Although the sample for this study was small (four teachers in three English schools) the qualitative data was fine-grained and gathered over the course of one year. Aubrey identified pedagogical subject knowledge through observation of classroom discourse – the representations teachers used, what they said, did or demonstrated, what the children said, did or showed, and from interviews. These observations in a 'real' setting contrast with both Ma and Ball's approaches which drew on task-focused interviews with the teachers. It is important to note these differences, although an extended discussion of measures of effectiveness is beyond the scope of this paper. Aubrey's criteria for teacher effectiveness were evidenced in qualitative descriptions of teachers' behaviours, with one of the teachers "[displaying] precisely those teaching behaviours associated in the effective instruction literature with higher achievement scores" (p 145) i.e.

- systematic presentations of new ideas;
- reflection on and communication of the results of children's investigations
- importance attached to explanations;
- making explicit links between different representations (verbal, concrete, numerical, pictorial);
- accessing prior knowledge and making links to new ideas (although this was not always successfully achieved in whole class episodes).

These behaviours, claims Aubrey, were possible because of the interaction between the beliefs about learning and strength of the subject matter knowledge held by this teacher, enabling her to coordinate and use both her own subject knowledge and the mathematical knowledge which these young pupils already had on school entry. Although Aubrey did use pre- and post-observation interviews with the children, she claims that it was not possible to make any link between the identified progress of pupils over the year and the teaching behaviours of the four teachers.

Pupils' progress (as measured by test score gains) was, however, associated with particular beliefs and the nature of knowledge held by the larger sample (90 teachers, 2000 pupils, 18 case study teachers) in the Effective Teachers of Numeracy Project (Askew et al., 1997) at King's College London.

Although the mathematical focus was narrower than Aubrey's, the progress of pupils measured by tests of numeracy before and after the study revealed that teachers who believed that all pupils could become numerate and who themselves had 'knowledge and awareness of conceptual connections between the areas which they taught' (p. 3) produced the highest numeracy gains in pupils. These so-called 'connectionist' teachers did not necessarily hold advanced qualifications in mathematics i.e. 'A' level or beyond, but they were more likely to have attended extended Continuing Professional Development (CPD) courses. Initial training was perceived to have little influence on effectiveness, raising questions about the amount of time devoted to mathematics on postgraduate certificate in education (PGCE) courses and the relative contribution of ITT and CPD to deepening understanding.

Doubt about the effectiveness of mathematics specialists had earlier been highlighted in one part of a study of primary PGCE trainees (Carré and Ernest, 1993) involved a comparison of the teaching of mathematics by the specialists, with that of the whole cohort. The 'specialist' trainee teachers were so designated on the basis of their choice of specialist training in the course, which did not necessarily reflect an advanced mathematics qualification. When classroom teaching performance was assessed, it emerged that "there is virtually nothing to distinguish mathematicians and others in teaching mathematics" (p. 161). The situation was very different in the teaching of music (Bennett and Turner-Bisset, 1993), where specialists were judged to perform at a higher level of competence than other students. This was in accordance with the researchers' expectations (p. 164), since music specialist students had been assessed as having a high level of music subject-matter knowledge relative to other students, while this was not the case for mathematics specialists *vis a vis* mathematics.

At the time when teacher training institutions were required to audit trainee teachers' subject knowledge, the SKIMA (Subject knowledge in Mathematics) collaborative group was established. The relationship between SMK and teaching performance in number as judged by observing tutors was investigated for cohorts of primary trainees over several years. Early findings that effective classroom teaching of elementary mathematics was associated with secure SMK at a level beyond the elementary curriculum was replicated over this period (Rowland, Barber, Heal and Martyn, 2003)

Discussion

Of all the elements of teacher knowledge discussed in this overview, the category of subject matter knowledge would seem to be the most contentious. Primary teachers in England and Wales are required to have a minimum qualification (the equivalent of a grade C at the GCSE examination taken at age 16) in order to gain entry to a course of teacher training, but this in itself cannot guarantee coverage or connectedness. The introduction of an audit with its associated list of content was one attempt to define the knowledge which should underpin successful primary teaching, but this was a relatively crude way of addressing the issue and has now been abandoned. Treating SMK as something which comes before the preparation for teaching and which may have to patched up in some way does not give the message that SMK is a work in progress.

Aubrey's model, which subsumes SMK within pedagogical subject knowledge would seem to be a good way forward, since it acknowledges the importance of teaching in the whole enterprise. After all, 'teachers use mathematical knowledge not so much for the doing of mathematics but rather for the teaching of mathematics.' (Hodgen, 2003, p106). Nonetheless, attention to SMK, would seem to be necessary within the consideration of pedagogy. This is not pedagogy in the general sense e.g. whether a three part lesson should be used or not, but in looking specifically at how the mathematics will be taught. It would also seem necessary to make sure that syntactic knowledge, as part of SMK, is not neglected at the expense of substantive knowledge. How should this be done in initial teacher training and in continued professional development, whilst attending to the affective dimension of the trainees' learning? Is there still a case for surfacing and challenging, and how should this be incorporated into an approach which emphasizes pedagogical subject knowledge?

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